



## Effect of Self Feedback on Mean-Field Coupled Oscillators: Revival and Quenching of Oscillations

Saumen Chakraborty

*Department of Physics, Bidhan Chandra College,  
Asansol-713303, West Bengal, India  
saumenbcc@gmail.com*

Manaj Dandapathak

*Department of Physics, Purulia Govt. Engineering College,  
Aghorpur-723103, West Bengal, India  
phymanoj@yahoo.co.in*

Saumendra Sankar De Sarkar

*Department of Physics, Raniganj Girls' College,  
Searsole Rajbari-713358, West Bengal, India  
ssdesarkar@gmail.com*

Received July 9, 2020; Revised October 15, 2020

Interaction among different units in a network of oscillators may often lead to quenching of oscillations and the importance of oscillation quenching can be found in controlling the dynamics of many real world systems. But there are also many real life phenomena where suppression of oscillation should be avoided for maintaining the sustained evolution of the system. In this work, we propose a self-feedback control scheme through which one is able to either achieve quenching or to retrieve the rhythmic behavior in a network of mean-field diffusively coupled systems. It is found that for proper choice of the strength of the control signal, the system converges to an oscillatory state from oscillation quenched state and for further increase of the strength the system enters in chaotic region. Moreover, reversal of the phase of the control signal can induce suppression of oscillation. Thus, the proposed modification offers a better control on the dynamics of the mean-field coupled system. In addition to this, a new transition phenomenon from inhomogeneous limit cycle (IHLC) to homogeneous limit cycle (HLC) through chaotic route has also been found in the modified system.

*Keywords:* Quenching; revival of oscillation; inhomogeneous limit cycle; homogeneous limit cycle; self-feedback.

### 1. Introduction


Studies on coupled oscillator models provide an effective paradigm to explore the rich variety of the collective behaviors exhibited by different interacting dynamical systems. Phase locking, synchronization and quenching of oscillation are few well emergent behaviors among those. Such complex collective behaviors are observed in coupled oscillators, depending upon the nature and the strength of their

coupling. Phase locking and synchronization may be viewed as a phase effect due to weak interaction of the oscillators. The phenomenon of oscillation quenching emerges as the amplitude effect, whereby individual oscillators cease to oscillate when coupled strongly.

Quenching (suppression) of oscillation is manifested in two different ways, namely, Amplitude Death (AD) and Oscillation Death (OD). AD refers

2150078-1

Saumendra Sankar De Sarkar

Probing the Jacobi shape transition in hot and rotating  $^{43}\text{Sc}$ Debasish Mondal,<sup>1,2</sup> Deepak Pandit,<sup>1,2</sup> S. Mukhopadhyay,<sup>1,2</sup> Surajit Pal,<sup>1</sup> Pratap Roy ,<sup>1,2</sup> Vitissha Suman,<sup>3</sup> Balaram Dey,<sup>4</sup> Srijit Bhattacharya,<sup>5</sup> A. De,<sup>6</sup> C. Bhattacharya,<sup>1,2</sup> and S. R. Banerjee<sup>1</sup><sup>1</sup>Variable Energy Cyclotron Centre, 1/AF-Bidhannagar, Kolkata-700064, India<sup>2</sup>Homi Bhabha National Institute, Training School Complex, Anushaktinagar, Mumbai 400094, India<sup>3</sup>Health Physics Division, Bhabha Atomic Research Centre, Mumbai 400094, India<sup>4</sup>Department of Physics, Bankura University, Bankura 722155, India<sup>5</sup>Department of Physics, Barasat Government College, Barasat, N 24 Parganas, Kolkata 700124, India<sup>6</sup>Department of Physics, Raniganj Girls' College, Raniganj 713358, India (Received 8 September 2020; accepted 28 October 2020; published 18 November 2020)

The evolution of the hot and rotating  $^{43}\text{Sc}$  nucleus to a highly deformed shape has been studied by measuring the high-energy  $\gamma$  rays from the decay of the giant dipole resonance. The compound nucleus was populated at two initial excitation energies and average angular momenta of  $\approx 26$  and  $31\hbar$  by using  $^{16}\text{O}$  beam of energies  $E_{\text{cm}} = 120$  and  $142$  MeV, respectively. The evaporated neutron energy spectra have been measured for proper determination of nuclear level density. The angular momentum has been determined by measuring the low-energy  $\gamma$ -ray multiplicities. The high-energy  $\gamma$ -ray and neutron spectra were analyzed simultaneously. At  $\langle J \rangle \approx 26\hbar$  a near-oblate shape is observed, whereas at  $\langle J \rangle \approx 31\hbar$  a sharp peak is observed at  $E_{\gamma} \approx 10$  MeV pointing towards the transition to the Jacobi shape with quadruple deformation parameter  $\beta \approx 0.7$ . The results have been corroborated by the theoretical calculations based on the rotating liquid drop model framework.

DOI: 10.1103/PhysRevC.102.051302

The atomic nucleus is a many-body quantum system governed by the strong interaction among the constituent nucleons. However, many properties of the nucleus can be described by a macroscopic-microscopic model where the nucleus is assumed to be a macroscopic charged liquid drop or a more sophisticated system with a microscopic shell effect [1–5]. Due to the shell effect, the nucleus mostly has a prolate shape in the ground state and at low excitations rotates collectively in which the axis of rotation remains perpendicular to the symmetry axis. The shell effect melts above a temperature ( $T$ ) of  $\approx 2.0$  MeV and a rotating nucleus assumes a noncollective oblate shape where the rotation axis coincides with the axis of symmetry. With the increase in angular momentum ( $J$ ), the nucleus becomes more oblate deformed, and above a critical angular momentum ( $J_c$ ), the shape changes to a highly deformed ( $\beta > 0.6$ ) triaxial or a nearly collective prolate shape called the Jacobi shape. This shape transition was first suggested by Beringer and Knox [6] based on the observation of gravitational rotating systems, which under the influence of high angular momentum, change their shape from the so-called Maclaurin spheroid to a Jacobi ellipsoid. Later, many authors have theoretically predicted the shape transition for rotating nuclei based on semiclassical models [7–10]. It is observed that for mass number  $A \gtrsim 180$ , the nucleus undergoes fission with  $J < J_c$ , the value of which depends on the nuclear mass and charge [8,9]. Therefore, one could only observe the Jacobi shape transition experimentally in light and

medium mass nuclei where  $J_c$  is much smaller than the critical angular momentum for fission.

Experimentally, the high-energy  $\gamma$ -ray line shape originating from the decay of the giant dipole resonance (GDR) [11,12] serves as an excellent tool to probe the shape of the nucleus at high  $T$  and  $J$ . Macroscopically, the GDR is described as the out-of-phase oscillation of proton and neutron fluids keeping a dipole shape. The resonance energy is inversely related to the axis length along which the vibration occurs, resulting in the splitting of the  $\gamma$ -ray line shape depending on the overall shape of the nucleus. The lifetime of the GDR being very small, it can also probe the shape of the nucleus at high  $T$  and  $J$ . However, the presence of thermal fluctuation at high excitations vitiates the signature of any shape transition [13–15] and it is difficult to draw any inference from only the  $\gamma$ -ray line shape. But, it has been shown, within the framework of the adiabatic thermal shape fluctuation model (TSPM) calculations, that the averaged absorption cross section gives a clear signature of the transition from a noncollective oblate to a highly deformed Jacobi shape in light mass nuclei [16]. It is also interesting to note that for light nuclei the rotational frequency for a given angular momentum is large as compared to that of heavy nuclei. This results in Coriolis splitting of the  $\gamma$ -ray line shape [17] and the three-component Lorentzian line shape for a collectively rotating deformed nucleus further splits into a five-component Lorentzian. The peak arising due to the vibration along the largest axis splits into two components, and a sharp peak is observed around  $E_{\gamma} \approx 10$  MeV which is the unambiguous signature of the transition from noncollective oblate to the Jacobi shape. Apart from this

\*debasishm@vecc.gov.in

Alok Kumar De



## SOME COMMON FIXED POINT THEOREM IN METRIC SPACES OF FISHER AND SESSA

KRISHNADHAN SARKAR

**ABSTRACT.** In this paper we have proved for two mappings that they have a unique common fixed point on a compact subset of a metric space. Where one of the mapping is non-expansive and the pair of mappings is weakly commuting. Fisher and Sessa [5] proved the same problem with a closed subset. We replaces the main results by change the closed subsets to compact subsets. Baboli and Ghaemi [1] proved the same type of theorem. We have extended the results of Baboli and Ghaemi [1]

### 1. INTRODUCTION

In 1922, Banach first introduced the concept of fixed point theorem in his Ph.D. thesis. He introduced the notion of contraction mapping, which is known as Banach contraction Principle. Later on Schauder proved a fixed point theorem as: If  $A$  is a compact, convex subset of a Banach space  $X$  and  $T : A \rightarrow A$  is a continuous function, then  $T$  has a fixed point. The compactness condition on  $A$  is a stronger condition and most of the problems in analysis do not have compact setting. In 1986, Fisher and Sessa [7] proved a fixed point theorem for two self maps on a subset of a Banach Space which is closed convex. Sessa [7] further generalized a results of Das and Naik [2]. In 2015, Baboli and Ghaemi [1] generalized the results of Fisher and Sessa [5].

### 2. PRELIMINARIES

Schauder further proved the following theorem without using the notion of compactness as:

**Theorem 2.1.** If  $A$  is a closed bounded convex subset of a Banach space  $X$  and  $T : A \rightarrow A$  is a continuous map such that  $T(A)$  is a compact, then  $T$  has a fixed point.

In this paper we generalize the results of Baboli and Ghaemi [1].

---

1991 *Mathematics Subject Classification.* 47H10, 54H25.

*Key words and phrases.* Common Fixed Point, Non-expansive mappings, Weakly commuting mappings.

Submitted Aug. 15, 2019. Revised Oct. 9, 2019.

Krishnadhan Sarkar

**SOME COMMON FIXED POINT RESULTS IN 2-BANACH SPACES**

**Krishnadhan Sarkar, Dinanath Barman\* and Kalishankar Tiwary\***

Department of Mathematics,  
Raniganj Girls' College,  
Raniganj, Paschim Bardhaman, West Bengal - 713358, INDIA

E-mail : sarkarkrishnadhan@gmail.com

\*Department of Mathematics,  
Raiganj University, West Bengal - 733134, INDIA

E-mail : dinanathbarman85@gmail.com, tiwarykalishankar@yahoo.com

(Received: Jan. 26, 2019 Accepted: Jun. 08, 2020 Published: Aug. 30, 2020)

**Abstract:** In this paper, we have proved some common fixed point theorems of a family of self maps without continuity in 2-Banach space. We have used functions on  $\mathbb{R}_+^5$  to  $\mathbb{R}_+$  and also generalize many existing results.

**Keywords and Phrases:** 2-norm, 2-Banach.

**2010 Mathematics Subject Classification:** 54H25, 47H10.

### 1. Introduction

In 1965, Gähler ([5], [6]) introduced 2-Banach space and Iseki [7] obtained some results on fixed point theorems in 2-Banach spaces. After the introduction of 2-Banach space many research workers have extended fixed point theorems of metric, Banach spaces etc. in the new setup of 2-Banach spaces. Mishra et al. [10], Khan and Khan [8], Saha et al. [12], Mishra et al. [11], Saluja [13], Saluja and Dhakde [14], Das et al. [1], Shrivastava [15], Das et al. [2] - [3], Liu et al. [9] and etc. have worked on fixed point and common fixed point theorems in this space. In this paper we also have proved some unique common fixed point theorems in 2-Banach spaces.

*Krishnadhan Sarkar*

**COMMON FIXED POINT THEOREMS USING T-HARDY  
ROGERS TYPE CONTRACTIVE CONDITION AND  
F-CONTRACTION ON A COMPLETE 2-METRIC SPACE**

DINANATH BARMAN, KRISHNADHAN SARKAR AND KALISHANKAR TIWARY

**ABSTRACT.** In this paper we have proved some common fixed point theorems using T-Hardy Rogers Type Contraction condition and F-Contraction on a complete 2-metric space and generalized many existing results in this literature.

1. INTRODUCTION

Fixed point is an important part of mathematics. It is used in various branches of mathematics such as Numerical Analysis, Differential Equation, Functional Analysis, Topology etc. Not only in Mathematics, it is also used in Biology, Chemistry, Physics and also in many other branches.

In 1922, Banach first investigate a fixed point theorem in metric space and it is well known as Banach Fixed Point Theorem. After that many researchers of this field have generalized that theorem in various ways. They have restricted the conditions or have changed the spaces. After Banach, Kannan [9] generalized that theorem. After Kannan, Chaterjea [4] generalized that fixed point theorem. Reich gave a generalization of Chaterjea's [4] fixed point theorem. In 1973, Hardy and Rogers [7] have also generalized the fixed point theorem of Reich. After that, many researchers have been using different type of Hardy Rogers contractive condition to obtain a new fixed point results.

In this paper we have generalized many existing results which are in 2-metric spaces. 2-metric space is a generalization of metric space, which has been introduced by Gähler [6]. In this space, the mapping goes  $X \times X \times X \rightarrow \mathbf{R}^+$ . If  $T$  is a mapping in 2-metric space then  $T : X \times X \times X \rightarrow \mathbf{R}^+$ . Basically, 2-metric means the area of a triangle in  $\mathbf{R}$ .

In this paper we have introduced T-Hardy Rogers contractive condition and F-contraction in 2-metric spaces.

M. Abbas et. al [1], Altun et. al [2] uses T- Hardy Rogers condition to prove fixed point theorem in various spaces other than 2-metric spaces.

2010 *Mathematics Subject Classification.* 47H10, 54H25.

*Key words and phrases.* Complete 2-metric space, T-Hardy Rogers contraction condition, F-contraction.

Submitted Sep. 19, 2019.

*Krishnadhan Sarkar*